What's wrong with ad hoc theory modifications?

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Ad hoc hypotheses

'Ad hoc' literally means *for this*.

An *ad hoc hypothesis* is commonly characterized as one that, in light of a certain observation, has been proposed merely in order to save a theory from refutation.

Since I do not wish to distinguish between hypotheses and theories, I will consider the following question:

What is wrong with, in light of observations contradicting one's hypotheses, modifying one's hypotheses in an ad hoc manner?

For starters: what does it mean?

A matter of taste?

Hunt 2012:

> *In this article I review attempts to define the term "ad hoc hypothesis" focusing on the efforts of, among others, Karl Popper, Jarrett Leplin, and Gerald Holton. I conclude that the term is unhelpful; what is "ad hoc" seems to be a judgment made by particular scientists not on the basis of any well-established definition but rather on their individual aesthetic senses. Further, a hypothesis considered ad hoc can apparently be retroactively declared non–ad hoc on the basis of subsequent data, rendering the term meaningless.*

Contrary to Hunt, I claim that the term 'ad hoc' can be understood in a way that explains its negative connotations.

Example

Hypothesis: All ravens are black.

Observation: Albin is a white raven.

Photo: Gregory Messimer

Ad hoc modification: All ravens are black except Albin.

Falsifiability

According to Popper, the ad hoc modified hypothesis is characterized by being empirically weaker (less falsifiable), and therein lies its flaw.

But the hypothesis can be ad hoc modified in different ways:

Weakening ad hoc modification: All ravens are black except (possibly) Albin.

Non-weakening ad hoc modification: All ravens are black except Albin who is white.

However, the non-weakened ad hoc modified hypothesis does not yield any *novel predictions*. It does imply that Albin is white, which we already know by now. Apart from that, it has the same empirical consequences as the original hypothesis.

Comparing empirical strength

How to compare the empirical strength of logically independent theories?

Suggestion: you can compare their empirical strength relative to a set of observation sentences.

A theory is said to be **complete** with respect to a set of sentences if, for each sentence in the set, the theory either entails the sentence or its negation.

Two logically independent theories can be said to have **the same empirical strength** relative to a set of observation sentences if they are both complete with respect to it.

Empirical states and observations

Let $\Sigma = \{S_0, S_1, S_2, \ldots\}$ be a countably infinite and completely logically independent set of observation sentences.

An **empirical state** is a function assigning a truth value to each sentence of Σ.

Every empirical state corresponds to an infinite sequence of **observations**:

- S_0 is true, S_1 is true, S_2 is true, S_3 is true, ...
- S_0 is false, S_1 is true, S_2 is true, S_3 is true, ...
- S_0 is false, S_1 is false, S_2 is true, S_3 is true, ...

$\vdots$

Guessing strategies

A **guessing strategy** if a function that, for every finite sequence of observations, returns a consistent theory that is complete with respect to Σ.

The following concept of success is essentially due to Putnam 1965 and Gold 1967:

A guessing strategy is said to be **successful in the limit** with respect to an empirical state if the strategy, after a finite number of observations, returns a theory consistent with the empirical state, and if it keeps returning the same theory for all subsequent observations.

A basic impossibility result

Theorem 1

For every guessing strategy, there is an empirical state with respect to which the strategy is not successful in the limit.

Proof.

Given an arbitrary guessing strategy, one can define an empirical state where the truth value of each observation sentence is the opposite of what the guessing strategy suggests. □

Hume's lesson: induction only works if nature is uniform.

The uniformity of nature

Based on the the classical definition of randomness due to Church 1940, I suggest the following characterization:

An empirical state can be said to be **uniform** if it is completely described by an axiomatizable theory.

Theorem 2

There is a guessing strategy that is successful in the limit with respect to every uniform empirical state.

A simple guessing strategy

Let $T_0, T_1, T_2, \ldots$ be an enumeration of all axiomatizable theories that are consistent and complete with respect to Σ.

- Start by guessing on T_0.
- If your current guess is on T, and the latest observation agrees with T, keep guessing on T. If the observation disagrees with T, guess on the first theory in the enumeration agreeing with your observations.

It is easy to show that this strategy is successful in the limit with respect to every uniform empirical state.

Radical and conservative guessing strategies

At this level of abstraction, is there anything corresponding to our intuitive idea of "modifying one's hypotheses in an ad hoc manner"?

Perhaps: to guess on a theory that differs from one's previous guess with respect to at most finitely many observation sentences.

A guessing strategy is said to be **conservative** if, each time it switches theory, switches to a theory that differs from the previous with respect to at most finitely many sentences in Σ.

A guessing strategy is said to be **radical** if, each time it switches theory, switches to a theory that differs from the previous with respect to infinitely many sentences in Σ.

The existence of radical guessing strategies

If we assume that Σ is not only enumerable, but *recursively* enumerable (i.e. is enumerated by a recursive/computable function), it is easy to establish the following:

Lemma 1

For every axiomatizable theory T that is consistent and complete with respect to Σ, and for every finite sequence of observations, there is an axiomatizable theory that is also consistent and complete with respect to Σ, that agrees with the observations, but differs from T with respect to infinitely many sentences in Σ.

Under the same assumptions, one can also establish the existence of conservative guessing strategies.

A radical guessing strategy

Assume that Σ is recursively enumerable, and let $T_0, T_1, T_2, \ldots$ be an enumeration of all axiomatizable theories that are consistent and complete with respect to Σ.

- Start by guessing on T_0.
- If your current guess is T, and your latest observation agrees with T, keep guessing on T. If the observation disagrees with T, guess on the first theory in the enumeration agreeing with all your observations, but differing from T with respect to infinitely many sentences in Σ. (The existence of such a theory is guaranteed by the previous lemma.)

It is easy to show that this strategy is also successful in the limit with respect to every uniform empirical state.

Proof

Suppose that T_n is the first theory in the enumeration agreeing with the actual empirical state. After finitely many observations, T_n will also be the first theory in the enumeration agreeing with all observations. At that point, either of two things can happen:

1. T_n differs from your current guess with respect to infinitely many sentences in Σ. Then your next guess will be T_n, and you will hold on to that guess forever after.
2. T_n does not so differ from your current guess. Then your next guess will not be T_n, but some other theory T, differing from T_n with respect to infinitely many sentences in Σ. But at some point, since T does not agree with the empirical state, an observation will be made disagreeing with T. Then your guess will be T_n, and you will hold on to that guess forever after.

In both cases, your strategy is successful in the limit.

The downside of conservative guessing strategies

Theorem 3

Assume that Σ is recursively enumerable. For every conservative guessing strategy, there is a uniform empirical state with respect to which the strategy is not successful in the limit.

Proof.

Suppose that T is the first guess of a conservative guessing strategy. By the previous lemma, there is an axiomatizable theory, with a corresponding uniform empirical state, differing from T with respect to infinitely many sentences in Σ. The conservative strategy will never guess on a theory agreeing with this state. $\square$

Conclusions

We have established the following purely technical result:

Theorem 4

Assume that Σ is recursively enumerable. Then there is a radical, but no conservative, guessing strategy that is successful in the limit with respect every uniform empirical state.

What, then, is wrong with modifying one's hypotheses in an ad hoc manner?

Short answer:

- *Sometimes* doing it may still lead to success.
- *Always* doing it may also lead to success, but does not guarantee it.
- *Never* doing it guarantees success.

Referenser

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